Long Question with Answer (7-8 marks)

Q① Explain scalar and vector potentials why are they not unique?

Ans:-

∵ Magnetic field

$$\vec{B} = \nabla \times \vec{A}$$

Electric field

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Potentials are not unique because we can perform guage transformation

$$\vec{A'} = \vec{A} + \nabla \psi$$

$$V' = V - \frac{\partial \psi}{\partial t}$$

without changing E and B.
This is called guage invariance.

Q② Prove that electromagnetic waves are transverse

Ans:- Assume plane wave.

$$\vec{E} = \vec{E_0} e^{i(kx - \omega t)}$$

Taking divergence

$$\vec{k} \cdot \vec{E_0} = 0$$

This means $\vec{E} \perp \vec{k}$

Similarly

$$\vec{B} \perp \vec{k} \qquad \text{So EM waves are transverse}$$

Q3) Derive expression for EM energy stored in capacitor

Ans. Energy density

$\Rightarrow$ $\qquad u = \frac{1}{2} \varepsilon_0 E^2$

$\Rightarrow$ Between parallel plates :-

$\Rightarrow$ $\qquad E = \frac{V}{d}$

∴ Total Energy

$\Rightarrow$ $\qquad U = \int u \, dv$

$\Rightarrow$ For plate area A :

$\Rightarrow$ $\qquad u = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2 Ad$

$\Rightarrow$ $\qquad = \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2$

∴ $\qquad C = \frac{\varepsilon_0 A}{d}$

∴ $\qquad \boxed{U = \frac{1}{2} CV^2}$